Math 520 Homework 1

1. a) Prove directly from the definition that e^z is differentiable at 0.
b) Prove directly from the definition that z^n is differentiable and $(z^n)' = nz^{n-1}$.

2. [Alhfors, p. 28 #2], *modified*: Verify Cauchy-Riemann's equations for the functions z^2, z^3, e^z, and the principal branch of $\log z$.

3. [Alhfors, p. 28 # 5] Prove that the functions $f(z)$ and $\overline{f(\bar{z})}$ are simultaneously analytic.

4. [Alhfors, p. 28 # 7] Prove that a harmonic funciton satisfies the formal differential equation

$$\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$$

5. [Alhfors, p. 72 #1] Give a precise definition of a single-valued branch of $\sqrt{1+z} + \sqrt{1-z}$.

6. Prove that $f(z) = \bar{z}$ is not differentiable at any point in $\mathbb{C}$.

7. [Conway, p. 43 #1] Prove that $f(z) = |z|^2$ has a derivative only at the origin.

8. Analize when $az + b\bar{z} + c = 0$, where $a, b, c \in \mathbb{C}$, represents a line, point, or an empty set.