

Math 520 Homework 10

Due: Monday, April 20, 2026, 7pm

The homework is optional—no credit.

The homework can be turned in as a pdf to the gradescope; code 5DN86V

1. [Ahlfors, p. 154, #1] How many roots does the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ have in the disk $|z| < 1$? Hint: Look for the biggest term when $|z| = 1$ and apply Rouché's theorem.

2. [Ahlfors, p. 154, #3] How many roots of the equation $z^4 + 8z^3 + 3z^2 + 8z + 3 = 0$ lie in the right half plane? Hint: Sketch the image of the imaginary axis and apply the argument principle to a large half disk.

In the rest of the exercises evaluate the integral by the method of residues:

3. [Ahlfors, p. 161, #3a]

$$\int_0^{\pi/2} \frac{dx}{a + \sin^2 x},$$

where $|a| > 1$.

4. [Ahlfors, p. 161, #3b]

$$\int_0^{\infty} \frac{x^2 dx}{x^4 + 5x^2 + 6}$$

5. [Ahlfors, p. 161, #3e]

$$\int_0^{\infty} \frac{\cos x}{x^2 + a^2} dx,$$

where a is real.

6. [Ahlfors, p. 161, #3g]

$$\int_0^{\infty} \frac{x^{1/3}}{1 + x^2} dx.$$

7. [Ahlfors, p. 87, #3h]

$$\int_0^{\infty} \frac{\log x}{1 + x^2} dx.$$