Math 520 Homework 3

Due: Monday, February 16, 2026
There will be penalty imposed for late homework.
The homework needs to be turned in as a pdf to the gradescope; code 5DN86V

Read [Ahlfors, pp. 89–99] on elementary conformal mappings.

1. [Ahlfors, p. 96 #1] Map the common part of the disks $|z| < 1$ and $|z - 1| < 1$ on the inside of the unit circle. Choose the mapping so that the two symmetries are preserved.

2. [Ahlfors, p. 96 #2] Map the region between $|z| = 1$ and $|z - 1/2| = 1/2$ on the half plane.

3. [Ahlfors, p. 97 #3] Map the complement of the arc $|z| = 1$, $y \geq 0$ on the outside of the unit circle so that the points at ∞ correspond to each other.

4. [Ahlfors, p. 97 #5] Map the outside of the right-hand branch of the hyperbola $x^2 - y^2 = a^2$ on the disk $|w| < 1$ so that the focus corresponds to $w = 0$ and the vertex to $w = -1$.

5. [Conway, p. 55 # 8] If $Tz = \frac{az+b}{cz+d}$, show that $T\mathbb{R}_\infty = \mathbb{R}_\infty$ if and only if we can choose a, b, c, d to be real numbers. (Here, $\mathbb{R}_\infty = \mathbb{R} \cup \{\infty\}$.)

6. [Conway, p. 55 # 9] If $Tz = \frac{az+b}{cz+d}$, find necessary and sufficient conditions that $T(\Gamma) = \Gamma$, where Γ is the unit circle $\{z : |z| = 1\}$.