Math 520 Homework 4

Due: Monday, February 23, 2026
The homework is optional—no credit.
The homework needs to be turned in as a pdf to the gradescope; code 5DN86V

1. [Ahlfors, p. 108 #1] Compute

$$\int_\gamma x\,dz$$

where γ is the directed line segment from 0 to $1+i$.

2. [Ahlfors, p. 108 #2] Compute

$$\int_{|z|=r} x\,dz$$

for the positive sense of the circle, in two ways: first, by use of a parameter, and second, by observing that $x = \frac{1}{2}(z+\bar{z}) = \frac{1}{2}\left(z + \frac{r^2}{z}\right)$ on the circle.

3. [Ahlfors, p. 108, #3] Compute

$$\int_{|z|=2} \frac{dz}{z^2-1}$$

4. [Ahlfors, p. 108, #4] Compute

$$\int_{|z|=1} |z-1|\,|dz|.$$

5. [Ahlfors, p. 108, #5] Suppose that $f(z)$ is analytic on a closed curve γ (i.e., f is analytic in a region that contains γ). Show that

$$\int_\gamma \overline{f(z)} f'(z)\,dz$$

is purely imaginary. (The continuity of $f'(z)$ is taken as granted.)

6. [Ahlfors, p. 108, #6] Assume that $f(z)$ is analytic and satisfies the inequality $|f(z)-1| < 1$ in a region Ω. Show that

$$\int_\gamma \frac{f'(z)}{f(z)}\,dz = 0$$

for every closed curve in Ω. (The continuity of $f'(z)$ is taken for granted).

7. [Ahlfors, p. 108, #7] If $P(z)$ is a polynomial and C denotes the circle $|z-a| = R$, what is the value of $\int_C P(z)\,d\bar{z}$? (Answer: $-2\pi i R^2 P'(a)$.)