Math 520 Homework 5

Due: Monday, March 2, 2026
The homework is optional—no credit.
The homework needs to be turned in as a pdf to the gradescope; code 5DN86V

1. [Ahlfors, p. 109, #8] Describe a set of circumstances under which the formula

$$\int_\gamma \log z \, dz = 0$$

is meaningful and true.

2. [Ahlfors, p. 117, #1] Give an alternate proof of Lemma 1 in [Ahlfors, p. 115] by dividing γ into a finite number of subarcs such that there exists a single-valued branch of $\arg(z-a)$ on each subarc. Pay particular attention to the compactness argument that is needed to prove the existence of such a subdivision.

3. [Ahlfors, p. 117, #2] It is possible to define $n(\gamma, a)$ for any continuous closed curve γ that does not pass through a, whether piecewise differentiable or not. For this purpose γ is divided into subarcs $\gamma_1, \ldots, \gamma_n$, each contained in a disk that does not include a. Let $\sigma = \sigma_1 + \cdots + \sigma_n$. We defined $n(\gamma, a)$ to the value of $n(\sigma, a)$. To justify the definition, prove the following:
(a) the result is independent of the subdivision;
(b) if γ is piecewise differentiable the new definition is equivalent to the old;
(c) the properties (i) and (ii) of the text ([Ahlfors, p. 116]) continue to hold.

4. [Ahlfors, p. 123, #2] Prove that a function which is analytic in the whole plane and satisfies an inequality

$$|f(z)| < |z|^n$$

for some n and all sufficiently large $|z|$ reduces to a polynomial.

5. [Ahlfors, p. 123, #3] If $f(z)$ is analytic and

$$|f(z)| \le M \quad \text{for } |z| \le R,$$

find an upper bound for

$$\left| f^{(n)}(z) \right|$$

when $|z| \le p < R$.

5a. Solve [Ahlfors, p. 118, #3] but no need to write it out or turn it in.