Math 520 Homework 6

Due: Monday, March 23, 2026, 7pm
The homework needs to be turned in as a pdf to the gradescope; code 5DN86V

1. [Ahlfors, p. 123, #1] Compute

$$\int_{|z|=1} e^z z^{-n}\,dz, \qquad \int_{|z|=2} z^n(1-z)^m\,dz \qquad \int_{|z|=\rho} |z-a|^{-4}|dz| \;\; (|a|\neq\rho).$$

where γ is the directed line segment from 0 to $1+i$.

2. [Ahlfors, p. 123, #5] Show that the successive derivatives of an analytic function at a point can never satisfy $f^{(n)}(z) > n!n^n$. Formulate a sharper theorem of the same kind.

3. [Ahlfors, p. 129, #1] If $f(z)$ and $g(z)$ have the algebraic orders h and k at $z=a$, show that fg has the order $h+k$, f/g the order $h-k$, and $f+g$ an order which does not exceed $\max\{h,k\}$.

4. [Ahlfors, p. 130, #2] Show that a function which is analytic in the whole plane and has a nonessential singularity at ∞ reduces to a polynomial.

5. [Ahlfors, p. 130, #4] Show that any function which is meromorphic in the extended plane is rational.

6. [Ahlfors, p. 130, #6] Show that an isolated singularity of $f(z)$ cannot be a pole of $\exp f(z)$. (Hint: f and e^f cannot have a common pole (why?). Now apply Theorem 9 on [A, p.129].)

7. [Ahlfors, p. 130, #3] Show that the functions e^z, $\sin z$, and $\cos z$ have essential singularities at ∞.