Math 520 Homework 9

Due: Monday, April 13, 2026, 7pm
The homework is optional—no credit.
The homework can be turned in as a pdf to the gradescope; code 5DN86V

1. [Ahlfors, p. 148, #1] Prove without use of Theorem 16 that $p dx + q\,dy$ is locally exact in Ω if and only if

$$\int_{\partial R} p\,dx + q\,dy = 0$$

for every rectangle $R \subset \Omega$ with sides parallel to the axes.

2. [Ahlfors, p. 148, #2] Prove that the region obtained from a simply connected region by removing m points has the connectivity $m + 1$, and find a homology basis.

3. [Ahlfors, p. 148, #4] Show that the single-valued analytic branches of $\log z$, z^α, and z^z can be defined in any simply connected region which does not contain the origin.

4. [Ahlfors, p. 148, #5] Show that a single-valued analytic branch of $\sqrt{1-z^2}$ can be defined in any region such that the points ± 1 are in the same component of the complement. What are the possible values of

$$\int \frac{dz}{\sqrt{1-z^2}}$$

over a closed curve in the region?

5. [Conway, p. 87, #1] Suppose $f\colon G \to G$ is analytic, and define $\phi\colon G \times G \to \mathbb{C}$ by $\phi(z, w) = (f(z) - f(w))/(z - w)$ if $z \neq w$ and $\phi(z, z) = f'(z)$. Prove that ϕ is continuous and for each fixed w, $z \mapsto \phi(z, w)$ is analytic.

6. [Conway, p. 87, #8] Let G be a region and suppose $f_n\colon G \to \mathbb{C}$ is analytic for each $n \geq 1$. Suppose that $\{f_n\}$ converges uniformly to a function $f\colon G \to \mathbb{C}$. Show that f is analytic.

7. [Conway, p. 87, #9] Show that if $f\colon \mathbb{C} \to \mathbb{C}$ is a continuous function such that f is analytic off $[-1, 1]$ then f is an entire function.