Lecture Summaries

Spring 2026, Math 520, Igor Kukavica

Lecture 1, Monday Jan 12, 2026, 11am: Power series. Power series for e^z, $\sin z$, $\cos z$, $\log(1+z)$, and $(1-z)^{-1}$. Radius of convergence. Product of two power series. Definition of a (complex) derivative. A function analytic in a set which is not necessarily open. Entire functions. Examples: polynomials, exponential, $sinz$, $\cos z$. Sums, differences, products, quotients of analytic functions. Chain rule (statement).

Lecture 2, Monday Jan 12, 2026, 2pm: Chain rule (proof). The derivative of an inverse. Logarithm (motivation). Logarithm. Branch of the logarithm. Principal branch of the logarithm.

Lecture 3, Wednesday Jan 14, 2026: The derivative of the logarithm. Cauchy-Riemann equations. The real and imaginary parts of an analytic function satisfy the Cauchy-Riemann equations. If $u, v \in C^1$ satisfy the Cauchy-Riemann equations, then $f = u + iv$ is analytic. The real and imaginary parts of an analytic function are harmonic.

Lecture 4, Wednesday Jan 21, 2026: Conjugate harmonic function, definition. Local existence of a harmonic conjugate. An analytic function such that either the derivative vanishes, or the image belongs to a line or a circle must be constant.

Lecture 5, Friday Jan 23, 2026: Definition of paths, smooth (C^1) paths, and piecewise smooth paths. The angle between two smooth curves. Analytic functions preserve the angles at points where the derivative is not zero. Example of a function ($\overline{z}$) which preserves the size of the angles but not the direction. A conformal mapping—the definition. A mapping which preserves angles and the derivative is not zero must be analytic. Complex form of the Cauchy-Riemann equations.

Lecture 6, Monday Jan 26, 2026: Linear fractional transformation (=linear transformation), Möbius transformation. Riemann sphere $\mathbb{C}_\infty$. Every linear transformation is a composition of a translation, inversion, rotation, dilation, and another translation, or a rotation, dilation, and translation. Mapping z_2, z_3, z_4 to $1, 0, \infty$ uniquely. Cross-ratio—a definition. A Möbius transformation preserves the cross-ratio. Assigned Homework #1 .

Lecture 7, Wednesday Jan 28, 2026: We have $(z_1, z_2, z_3, z_4) \in \mathbb{R}$ iff z_1, z_2, z_3, z_4 belong to the same circle. Corollary: Finding a mapping which takes z_1, z_2, z_3 to w_1, w_2, w_3. Corollary: A Möbius transformation maps circles to circles. Definition of symmetric points with respect to a circle. The symmetry definition does not depend on the choice of points on the circle; also the symmetry point is unique.

Lecture 8, Friday Jan 30, 2026: Geometric meaning of symmetry with respect to lines and circles. Conformal mappings. Conformal mapping of a half space (upper and right). Mapping properties of $(z-i)/(z+i)$. Mapping properties of $(z-a)/(z+b)$; Apollonius circles. Definition of $\int_\gamma f(z)\,dx$, $\int_\gamma f(z)\,dy$, $\int_\gamma f(z)\,dz$, and $\int_\gamma f(z)\,\overline{dz}$ for piecewise γ.

Lecture 9, Monday Feb 2, 2026: Arc-length integral. A necessary condition for a differential to be exact is that it has a primitive. A complex differential $f(z)\,dx$ is exact iff f has a primitive, which is an analytic function.

Lecture 10, Wednesday February 4, 2026: Integral of $(z-a)^n$ for $n \in \mathbb{N}$ over a closed curve. Integral of $(z-a)^m$ over a circle for $m \in \mathbb{Z}$. Cauchy's Theorem for a rectangle. Cauchy's Theorem for a rectangle for functions analytic except for a finite number of points where the function is not too singular.

Lecture 11, Friday February 6, 2026: For f analytic in a disk, the integral over any closed piecewise smooth curve vanishes. The same theorem, but allowing a finite number of points where the function is not too singular. Lemma on the integral of $1/(z-a)$. Index (winding number) of a curve around a point. Definition of the index when the curve is not closed.

Lecture 12, Monday February 9, 2026: Lemma on the integral of $1/(z-a)$—proof. Index of a curve which belongs to a disk around the points outside of the disk. Regions determined by a closed curve. The index is constant in the components determined by a closed curve. A criterion for index to be 1 (lemma to help with the Jordan curve theorem)—statement.

Lecture 13, Wednesday February 11, 2026: Proof of the criterion. Sketch of the proof of a weaker version of the Jordan curve theorem. A local Cauchy integral formula. Analytic functions are infinitely differentiable—statement.

Lecture 15, Friday February 13, 2026: